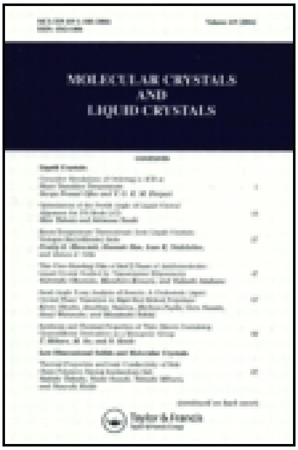

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Wave Propagation in Layered-Inhomogeneous Planar Anisotropic Media: Geometrical-Optics Approximation and Its Application

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WAVE PROPAGATION IN LAYERED-INHOMOGENEOUS PLANAR ANISOTROPIC MEDIA: GEOMETRICAL-OPTICS APPROXIMATION AND ITS APPLICATION

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Abstract The geometrical-optics approximation for describing the propagation of extraordinary waves in layered-inhomogeneous planar anisotropic media can be expressed as a series of laws. The results are applied to study the optical properties of spatially inhomogeneous nematic liquid crystal structures.

I. GEOMETRY

Approximation solutions, such as the geometrical-optics approximation (GOA), the phase-integral method and the method of perturbation theory, are of significance in the study of wave propagation in inhomogeneous media.[1] It has been shown that the GOA is an asymptotic solution of Maxwell's equations and for slowly varying inhomogeneous isotropic media, the GOA and its extension are most suitable.[1,2]

We studied the GOA for finding the approximation solutions for the electromagnetic fields in a layered-inhomogeneous planar anisotropic structure, in which the optical axis always lies in the *xz* plane of a Cartesian coordinate system and varies in the *z* direction. We let $\theta(z)$ be the angle between the optical axis and the *z* axis. Then the orientation of the optical axis can be described by the unit vector $\mathbf{n}(z) = (\sin\theta, 0, \cos\theta)$, and the components of the dielectric tensor is given by $\varepsilon_{ij}(z) = \varepsilon_\perp\delta_{ij} + (\varepsilon_\parallel - \varepsilon_\perp)\, n_i(z)n_j(z)$, where $\varepsilon_\perp = n_0^2$, $\varepsilon_\parallel = n_e^2$, n_0 and n_e are the ordinary and extraordinary refraction indexes. An electromagnetic wave propagating in the positive *z* direction is obliquely incident on the medium with polarization parallel to the plane of incidence, which is the xz plane. Consequently, only the extraordinary wave is excited in the medium.

II. LAWS OF GEOMETRICAL-OPTICS APPROXIMATION

In the GOA, the electromagnetic fields of frequency ω are expressed as

$$\exp\left[ik\phi(z) + ikpx - i\omega t\right] \sum_{n=0}^{n=\infty} k^{-n} F^{(n)}(z),$$

where $k = \omega/c$, and $p = \sqrt{\varepsilon_0}\, \sin\theta_0 \cos\phi_0$, depending on the angle of incidence θ_0, the azimuthal angle of the plane of incidence ϕ_0, and the dielectric constant ε_0 for the medium from which the wave is incident. Since the wave is polarized in the plane containing the optical axis, $\phi_0 = 0$ or π. The phase $k\phi(z)$ is almost linear in z and varies much more rapidly than the amplitudes $F^{(n)}(z)$, which are slowly varying functions of z. We obtained the zeroth-order and first-order GOA solutions and hence the GOA validation criterion.[3,4] By considering the conditions for which the GOA is an exact solution, we also obtained a theorem of nonreflecting structure in layered-inhomogeneous uniaxial media.[5] The results and their extensions are summarized as the following GOA laws and theorem.

1. GOA Law of Zeroth-Order Solution

The solution of the electromagnetic fields in the zeroth-order GOA are

$$E_x(\bar{x}) = A(\varepsilon_{33} - p^2)^{1/4} \exp\left[ik\phi(z) + ikpx\right],$$

$$E_z(\bar{x}) = R(z)\, E_x(\bar{x}),$$

$$H_y(\bar{x}) = n_0\, n_e\, E_x(\bar{x}) \,/\, \sqrt{\varepsilon_{33} - p^2},$$

where E_x is defined up to a constant, $\phi(z)$ and $R(z)$ are given respectively by

$$\phi = \pm \int_{Z_0}^{Z} \left[(n_0\, n_e \sqrt{\varepsilon_{33} - p^2} \mp p\, \varepsilon_{13})/\varepsilon_{33} \right] dz,$$

$$R = -\left(\varepsilon_{13} \pm p\, n_0\, n_e \,/\, \sqrt{\varepsilon_{33} - p^2}\right)/\varepsilon_{33},$$

A is a constant, z_0 is some constant, the upper sign is for wave propagating in the positive z direction and the lower sign is for wave propagating in the negative z direction.

2. GOA Law of First-Order Solution

The solution for the electromagnetic fields up to and including the first-order GOA are

$$E_x(\bar{x}) = A\left(F_1^{(0)} + \frac{1}{k}F_1^{(1)}\right)\exp\left[ik\phi(z) + ikpx\right],$$

$$E_z(\bar{x}) = A\,R(z)\left[F_1^{(0)} + \frac{1}{k}\left(pBF_1^{(0)} + F_1^{(1)}\right)\right]\exp\left[ik\phi(z) + ikpx\right],$$

$$H_y(\bar{x}) = \frac{A\,n_0\,n_e}{\sqrt{\varepsilon_{33} - p^2}}\left[F_1^{(0)} + \frac{1}{k}\left(F_1^{(1)} + QF_1^{(0)}\right)\right]\exp\left[ik\phi(z) + ikpx\right],$$

where

$$F_1^{(0)} = (\varepsilon_{33} - p^2)^{1/4}, \qquad F_1^{(1)} = (\varepsilon_{33} - p^2)^{1/4}\int_{z_0}^{z} M\,(\varepsilon_{33} - p^2)^{1/4}\,dz,$$

$$F_2^{(1)} = R(z)\left(p\,B\,F_1^{(0)} + F_1^{(1)}\right), \quad B = i\frac{1}{4\,R(z)\,(\varepsilon_{33} - p^2)^2}\frac{d\varepsilon_{33}}{dz},$$

$$Q = -i\frac{1}{4\,n_0\,n_e}\frac{\varepsilon_{33}}{(\varepsilon_{33} - p^2)^{3/2}}\frac{d\varepsilon_{33}}{dz},$$

$$M = \pm i\frac{1}{2n_0 n_e}\left[\frac{d^2F_1^{(0)}}{dz^2}\left(\frac{\varepsilon_{33}}{\varepsilon_{33} - p^2}\right) - \frac{p^2}{(\varepsilon_{33} - p^2)^2}\frac{dF_1^{(0)}}{dz}\frac{d\varepsilon_{33}}{dz}\right].$$

3. GOA Law of Validation Integral Criterion

The integral condition for the zeroth-order GOA to be valid is given by

$$\frac{1}{8k\,n_0\,n_e}\left|\int_{z_0}^{z}\left[\frac{1}{(\varepsilon_{33} - p^2)^{1/2}}\frac{d^2\varepsilon_{33}}{dz^2} + \frac{p^2}{(\varepsilon_{33} - p^2)^{3/2}}\frac{d^2\varepsilon_{33}}{dz^2}\right.\right.$$
$$\left.\left. - \frac{3}{4(\varepsilon_{33} - p^2)^{3/2}}\left(\frac{d\varepsilon_{33}}{dz}\right)^2 - \frac{p^2}{4(\varepsilon_{33} - p^2)^{5/2}}\left(\frac{d\varepsilon_{33}}{dz}\right)^2\right]dz\right| << 1.$$

4. GOA Law of Validation Sufficiency Criterion

The sufficient (but not necessary) condition for the GOA to be valid when the optical axis varies monotonically in the range (z_0, z) or (z, z_0) is

$$\frac{\varepsilon_{33}}{4\,k\,n_0\,n_e}\,\left|\,\frac{d}{dz}\,\frac{1}{\sqrt{\varepsilon_{33}-p^2}}\,\right| << 1.$$

5. GOA Law regarding Poynting Vector and Wave Vector

(a). The Poynting vector in a non-absorbing medium is given by $\mathbf{S}\,(z) = (cn_0n_e\,|\,A\,|^2/8\pi)\,\left[(\varepsilon_{13}+pn_0n_e/\sqrt{\varepsilon_{33}-p^2})/\varepsilon_{33}\,,\ 0\,,\ 1\,\right]$. The angle between the ray propagation direction and the z axis, $\theta_S(z)$, is given by $\tan\,\theta_S(z) = \left(\varepsilon_{13}+pn_0n_e/\sqrt{\varepsilon_{33}-p^2}\right)/\varepsilon_{33}$.

(b). The wave vector in the medium is given by $\mathbf{K}\ (z) = k\left[p,\ 0,\ (-p\varepsilon_{13}+n_0n_e\sqrt{\varepsilon_{33}-p^2})/\varepsilon_{33}\right]$. The angle between the wave propagation direction and the z axis, $\theta_K(z)$, is given by $\tan\theta_K(z) = p\varepsilon_{33}/\left(-p\varepsilon_{13}+n_0n_e\sqrt{\varepsilon_{33}-p^2}\right)$.

(c). The wave and the ray propagation directions satisfy the relation $\tan[\theta_S(z)-\theta(z)] = (n_0/n_e)^2\ \tan[\theta_K(z)-\theta(z)]$.

6. GOA Law of Effective Refractive Index and Snell Law

The effective refractive index is given by $n_{\mathrm{eff}}(z) = n_0\,n_e/\sqrt{n_0^2\ \sin^2(\theta-\theta_K)+n_e^2\ \cos^2(\theta-\theta_K)}$; and satisfies the Snell Law: $\sqrt{\varepsilon_0}\ \sin\ \theta_0\ \cos\ \phi_0 = n_{\mathrm{eff}}(z)\ \sin\ \theta_K(z)$.

7. Theorem of nonreflecting structure in layered-inhomogeneous uniaxial media

If the orientation of the optical axis in a layered-inhomogeneous uniaxial medium follows

$$\cos^2\theta(z) = \frac{\varepsilon_\perp}{\varepsilon_\parallel-\varepsilon_\perp}\left[\frac{\varepsilon_\parallel}{u^2+\dfrac{1}{2k^2}\left(\dfrac{u''}{u}-\dfrac{3u'^2}{2u^2}\right)}-1\right]$$

with an arbitrary continuous function $u(z)$ which satisfies the following constraint $\min\,(\varepsilon_\perp,\varepsilon_\parallel) \le [u^2+(u''/u-3u'^2/2u^2)/2k^2] \le \max\,(\varepsilon_\perp,\varepsilon_\parallel)$, and becomes constant at infinity, then there is no reflected wave for a normally incident extraordinary wave with frequency $\omega = ck$; where $u' = du/dz$, $u'' = d^2u/dz^2$, min (a, b) and max (a, b) denote respectively the minimum and the maximum values of a and b.

III. APPLICATIONS OF THE GEOMETRICAL-OPTICS APPROXIMATION

The GOA has been used to study the optical-field-induced molecular reorientation and bistability in nematic liquid crystals (NLCs),[6,7] wave propagation in a periodically bent NLC[3,8] and a hybrid oriented NLC cell.[4] As a comparison between the GOA and exact solutions, we have considered the wave propagation in a periodically bent NLC and showed that within the range of the criterion for validation of the GOA, excellent agreement between the GOA and exact solutions is obtained.[3,8] For example, the difference between the GOA and the exact solutions for the fields are less than 2×10^{-5} of the values of the respectively exact solutions. We also applied GOA to study the electro-optical properties of a spatially inhomogeneous absorbing planar anisotropic NLC medium, and excellent agreement between the theoretical calculation and the experimental data was demonstrated.[9]

REFERENCES

1. L. M. Brekhovskikh, Waves in Layered Media, 2nd ed., trans. by O. Liberman (Academic, New York, 1980).
2. R. K. Luneberg, Propagation of Electromagnetic Waves (New York University, New York, New York, 1948).
3. H. L. Ong and R. B. Meyer, J. Opt. Soc. Am. A, 2, p. 198 (1985).
4. H. L. Ong, Phys. Rev. A, 32, p. 1098 (1985).
5. H. L. Ong, Optics Comm., 55, p. 1 (1985).
6. H. L. Ong, Phys. Rev. A, 28, p. 2393 (1983).
7. N. V. Tabiryan, A. V. Sukhov and B. Ya. Zel'dovich, Mol. Cryst. Liq. Cryst., 136, p. 1 (1986).
8. H. L. Ong and R. B. Meyer, J. Opt. Soc. Am., 73, p. 167 (1983).
9. H. L. Ong, Modelling of Guest-Host Nematic Liquid Crystal Displays (in preparation).